



Instructor: B.L. Daku  
Time: 10 minutes  
Aids: None

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Name:  
Student Number:

1. Determine the constant  $\beta$ , where

$$\beta = \frac{\cos(\omega_0(n+1)) + \cos(\omega_0(n-1))}{\cos(\omega_0 n)}.$$

What could this expression be used for if you were given three consecutive samples of a discrete-time signal?

to get  $\omega_0$

$$= \frac{\frac{1}{2} (e^{j\omega_0(n+1)} + e^{-j\omega_0(n+1)}) + \frac{1}{2} (e^{j\omega_0(n-1)} + e^{-j\omega_0(n-1)})}{\frac{1}{2} (e^{j\omega_0 n} + e^{-j\omega_0 n})}$$

$$= \frac{e^{j\omega_0 n} e^{j\omega_0} + e^{-j\omega_0 n} e^{-j\omega_0} + e^{j\omega_0 n} e^{-j\omega_0} + e^{-j\omega_0 n} e^{j\omega_0}}{e^{j\omega_0 n} + e^{-j\omega_0 n}}$$

$$= \frac{e^{j\omega_0 n} [e^{j\omega_0} + e^{-j\omega_0 n} e^{-j\omega_0} + e^{-j\omega_0} + e^{-j\omega_0 n} e^{j\omega_0}]}{e^{j\omega_0 n} + e^{-j\omega_0 n}}$$

$$= e^{j\omega_0 n} [1 + e^{-j2\omega_0 n}]$$

$$= e^{j\omega_0} \left[ 1 + \frac{-j2\omega_0 n}{e^{-j\omega_0 n}} \right] + e^{-j\omega_0 n} \left[ 1 + \frac{j2\omega_0 n}{e^{j\omega_0 n}} \right]$$

~~$$= [1 + e^{-j2\omega_0 n}]$$~~

$$= e^{j\omega_0} + e^{-j\omega_0} = \boxed{2 \cos(\omega_0) = \beta}$$